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IMPREGNATING A HEATED FILLER WITH A NON-NEWTONIAN FLUID

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An approximate parametric method is used to solve the planar temperature-dependent problem of continuously impregnating a heated filler with a fluid that has a power-law non-Newtonian viscosity.

Many composite materials are made by impregnating porous materials (fillers) with various fluids (binders), which than are polymerized or crystallized into a solid. The most convenient method to accelerate this process is to preheat the filler, which significantly reduces the viscosity of the binder during the impregnation. Here the fluid is held at a high temperature for only a short time, with no danger of thermal decomposition. An exact self-similar solution has been obtained [1] to the problem of using an ordinary viscous fluid for continuously impregnating a heated layer, which is drawn through a heated chamber. Because binders used in practice (resins and polymer melts) have more complex rheological properties, whose permeability differs from Darcy's law, the problem has been generalized [2, 3] to viscoplastic binders. The permeability is described by a generalized Darcy's law [4] for a linear temperature dependence of the rheological properties. An approximate parametric method was suggested to solve this (nonself-similar) problem. The method uses a cubic trinomial for the temperature profile. Here we examine an analogous problem of a power filtration law [5] for arbitrary temperature-dependence of the non-Newtonian viscosity and for more general heat-transfer boundary conditions at the surface of the filler. We also use a parametric method, but with a different representation of the temperature profile, which allows us to obtain the solution in a compact form suitable for numerical computations. The problem is solved analytically in the particular cases of small and large pressure gradients, and also for weak temperature dependence of the non-Newtonian viscosity.

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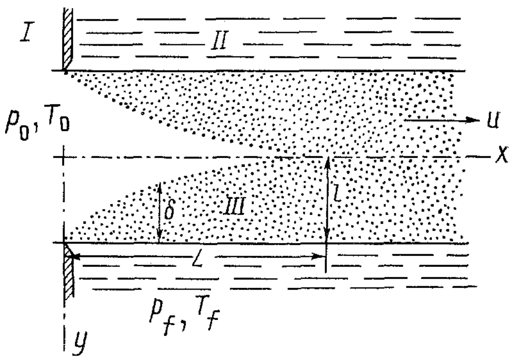

Fig. 1. Model of the impregnating apparatus.

Figure 1 shows a model of a continuous-impregnation apparatus. The filler, in the form of a thin layer, is heated in chamber I to a temperature T_0 at a pressure p_0 and drawn with a constant velocity u into chamber II, which is filled with a binder at temperature $T_f < T_0$ and pressure $p_f \geq p_0$. At each point of the impregnated part of the filler in region III, both phases are assumed to be at the same temperature, and the velocity of the binder along the x-axis coincides with the drawing velocity. Under these conditions, the equation for thermal conductivity takes the form [1]

$$u\frac{\partial\theta}{\partial x} + \varkappa v_y \frac{\partial\theta}{\partial y} = a\frac{\partial^2\theta}{\partial y^2}, \tag{1}$$

where

$$\theta = \frac{T - T_f}{T_0 - T_f};\quad \varkappa = \frac{\rho_f c_f}{\varepsilon\rho_f c_f + (1-\varepsilon)\rho_s c_s};\quad a = \frac{\lambda\varkappa}{\rho_f c_f}.$$

From the continuity equation for an incompressible fluid, it follows that the impregnation velocity v_y depends only on x. It is related to the width δ of region III by the obvious kinematic equation

$$v_y = -\varepsilon u\frac{d\delta}{dx}. \tag{2}$$

We will assume that the binder is a fluid with a power filtration law:

$$v_y = -\frac{1}{q}\left(\frac{\partial p}{\partial y}\right)^n, \tag{3}$$

in which the coefficient q (proportional to the non-Newtonian viscosity) is an arbitrary decreasing function of temperature:

$$q = q_f\zeta(\theta),\ \zeta(0) = 1. \tag{4}$$

The temperature dependence of the exponent n is usually rather small. Thus, according to [5], if pure vapor oil is heated from 28°C to 38°C, q changes by 77%, but n by 28%. If benzene with 82% resin is heated from 19°C to 26°C, q decreases by 37%, but n by 15%. Here we will neglect the temperature-dependence of n.

For boundary conditions, we use

$$p(x, l) = p_f,\ p(x, l-\delta) = p_0 - p_c,$$
$$\theta(x, l) = -\frac{1}{h}\frac{\partial\theta}{\partial y}(x, l),\ \theta(x, l-\delta) = 1. \tag{5}$$

The goal of this problem is to determine the working length L of the apparatus as a function of the impregnation parameters and the physical characteristics of the filler and binder.

If we use new independent variables

$$\delta = \delta(x), \ \eta = \frac{1}{\delta}(l - y), \tag{6}$$

Eqs. (1) and (3) take the form

$$\delta \frac{\partial\theta}{\partial\delta} - (\eta - \varkappa\varepsilon)\frac{\partial\theta}{\partial\eta} = \frac{1}{\omega}\frac{\partial^2\theta}{\partial\eta^2},$$
$$\omega = -\frac{v_y\delta}{\varepsilon a} = \frac{1}{\varepsilon a q}\left(-\frac{\partial p}{\partial\eta}\right)^n \delta^{1-n}. \tag{7}$$

Integrating these equations with respect to η from 0 to 1 yields

$$\delta^* \frac{df}{d\delta^*} + f = 1 - \varkappa\varepsilon\,[1 - \theta(\delta^*, 0)] + \frac{1}{\omega}\left[\frac{\partial\theta}{\partial\eta}(\delta^*, 1) - \frac{\partial\theta}{\partial\eta}(\delta^*, 0)\right], \tag{8}$$

$$\omega = \frac{\Omega}{\mu}\delta^{*1-n}. \tag{9}$$

Here

$$f = \int_0^1 \theta d\eta, \ \ \mu = \left[\int_0^1 \zeta^{1/n}(\theta)\, d\eta\right]^n,$$
$$\delta^* = \frac{\delta}{l}, \ \ \Omega = \frac{(p_f + p_c - p_0)^n}{\varepsilon a q_f l^{n-1}}. \tag{10}$$

We express the function $\theta(\delta^*, \eta)$ in the form

$$\theta = \theta_1 + \theta_2, \tag{11}$$

where θ_1 satisfies the equation

$$\frac{\partial^2\theta}{\partial\eta^2} + \omega(\eta - \varkappa\varepsilon)\frac{\partial\theta}{\partial\eta} = 0 \tag{12}$$

and the boundary conditions [5]:

$$\theta_1 = \frac{1}{a_1}\left\{\int_0^\eta \exp\left[-\frac{\omega}{2}(\eta - \varkappa\varepsilon)^2\right] d\eta + \frac{1}{hl\delta^*}\exp\left[-\frac{\omega}{2}(\varkappa\varepsilon)^2\right]\right\},$$
$$a_1 = \int_0^1 \exp\left[-\frac{\omega}{2}(\eta - \varkappa\varepsilon)^2\right] d\eta + \frac{1}{hl\delta^*}\exp\left[-\frac{\omega}{2}(\varkappa\varepsilon)^2\right]. \tag{13}$$

The function θ_2 is defined such that it vanishes both boundaries along with its first and second derivatives with respect to η; it also satisfies the integral equation derived from [11]:

$$\int_0^1 \theta_2 d\eta = f - f_1, \ \ f_1 = \int_0^1 \theta_1 d\eta. \tag{14}$$

According to (12) and (13):

$$f_1 = 1 - \varkappa\varepsilon\,[1 - \theta_1(\delta^*, 0)] + \frac{1}{\omega}\left[\frac{\partial\theta_1}{\partial\eta}(\delta^*, 1) - \frac{\partial\theta_1}{\partial\eta}(\delta^*, 0)\right] =$$
$$= 1 - \varkappa\varepsilon + \frac{1}{a_1}\left\{\frac{1}{\omega}\exp\left[-\frac{\omega}{2}(1 - \varkappa\varepsilon)^2\right] - \left(\frac{1}{\omega} - \frac{\varkappa\varepsilon}{hl\delta^*}\right)\exp\left[-\frac{\omega}{2}(\varkappa\varepsilon)^2\right]\right\}. \tag{15}$$

These requirements are satisfied by the expression

$$\theta_2 = 140(f - f_1)\,\eta^3(1 - \eta)^3. \tag{16}$$

When (15) is considered, Eq. (8) takes the form

$$\delta^* \frac{df}{d\delta^*} + f = f_1, \tag{17}$$

from which it follows that

$$f = \frac{1}{\delta^*} \int_0^{\delta^*} f_1 d\delta^* \tag{18}$$

because f and f_1 are bounded as $\delta^* \to 0$.

Thus chosen, θ satisfies Eq. (8), conditions (5), and an additional boundary condition for η = 1:

$$\frac{\partial^2 \theta}{\partial \eta^2} + \omega (1 - \varkappa \varepsilon) \frac{\partial \theta}{\partial \eta} = 0, \tag{19}$$

which follows from (7) and the last condition [5).

We integrate (2) with the conditions (9) and find the working length of the apparatus:

$$L = L_f L^*, \quad L_f = \frac{l}{\alpha \Omega (1 + n)},$$
$$L^* = (1 + n) \int_0^1 \mu \delta^{*n} d\delta^*, \quad \alpha = \frac{a}{lu}. \tag{20}$$

The function $\mu(\delta^*)$ is bounded by values which correspond to isothermal impregnation for $T = T_0$ and $T = T_f$:

$$\zeta(1) \leqslant \mu \leqslant 1 \tag{21}$$

$\mu(\delta^*)$ is determined by Eqs. (10)-(18) after specifying the function $\zeta(\theta)$.

If the binder is an ordinary viscous fluid (n = 1) and $h = \infty$, we obtain the exact self-similar solution [1]:

$$\theta = \theta_1(\eta), \quad f = f_1 = \text{const}, \quad \omega = \text{const},$$
$$\mu = \int_0^1 \zeta(\theta)\, d\eta = \text{const}, \quad L = \frac{l\mu}{2\alpha\Omega}. \tag{22}$$

We examine the case for arbitrary n under the condition $\Omega \sim 1/hl \ll 1$.

To first order, we obtain

$$\theta_1 = \eta + \frac{\omega_0}{6} \eta (1 - \eta)(1 - 3\varkappa\varepsilon + \eta) + \frac{1}{hl\delta^*}(1 - \eta),$$
$$f_1 = \frac{1}{2} + \frac{\omega_0}{24}(1 - 2\varkappa\varepsilon) + \frac{1}{2hl\delta^*}. \tag{23}$$

These expressions are valid in the interval $\delta_0 \leq \delta^* \leq 1$. For a small initial range $\delta^* \leq \delta_0$, the function f_1 can be considered constant and equal to its initial value f_0. According to (15)

$$f_0 = \begin{cases} 1 & \text{for } n \leqslant 1, \\ 1 - \varkappa\varepsilon & \text{for } n > 1. \end{cases} \tag{24}$$

From the continuity condition $f_1(\delta_0) = f_0$, we obtain an equation to determine δ_0:

$$(2f_0 - 1)\,\delta_0 = \frac{\Omega}{12\mu_0}(1 - 2\varkappa\varepsilon)\,\delta_0^{2-n} + \frac{1}{hl}, \tag{25}$$

from which it follows

$$\delta_0 = \begin{cases} \dfrac{1}{hl(2f_0 - 1)} & \text{for } n < 2, \\ \dfrac{\Omega}{12\mu_0} + \dfrac{1}{hl(1 - 2\varkappa\varepsilon)} & \text{for } n = 2, \\ \left(\dfrac{\Omega}{12\mu_0}\right)^{1/(n-1)} & \text{for } n > 2. \end{cases} \tag{26}$$

Thus:

$$f=\frac{1}{2}+\frac{1}{2}(2f_0-1)\frac{\delta_0}{\delta^*}+\frac{\omega_0}{24}(1-2\varkappa\varepsilon)\nu+\frac{1}{2hl\delta^*}\ln\frac{\delta^*}{\delta_0},$$

$$\mu=\mu_0\left\{1-210\,n(2f_0-1)\gamma\frac{\delta_0}{\delta^*}-\frac{1}{2}\omega_0 n\,[\beta-\right.$$

$$\left.-35(1-2\varkappa\varepsilon)\gamma(1-\nu)]-\frac{n}{hl\delta^*}\left[\frac{1}{\mu_0^{1/n}}-1-210\,\gamma\left(1-\ln\frac{\delta^*}{\delta_0}\right)\right]\right\},$$

$$\omega_0=\frac{\Omega}{\mu_0}\delta^{*1-n},\ \mu_0=\left[\int_0^1\zeta^{1/n}(\eta)\,d\eta\right]^n,$$

$$\beta=\frac{1}{3}-\varkappa\varepsilon+\frac{1}{\mu_0^{1/n}}\int_0^1\zeta^{1/n}(\eta)(2\varkappa\varepsilon-\eta)\,\eta d\eta, \tag{27}$$

$$\gamma=\frac{1}{\mu_0^{1/n}}\int_0^1\zeta^{1/n}(\eta)\,\eta^2(1-\eta)^2(1-2\eta)\,d\eta,$$

$$\nu=\begin{cases}\dfrac{1}{2-n}\left[1-\left(\dfrac{\delta_0}{\delta^*}\right)^{2-n}\right] & \text{for}\quad n\neq 2,\\[2ex] \ln\dfrac{\delta^*}{\delta_0} & \text{for}\quad n=2\end{cases}$$

and the coefficient L* takes the form

$$L^*=\mu_0-\frac{1}{4}n(1+n)\Omega[\beta-35(1-n)(1-2\varkappa\varepsilon)\gamma\nu_1]-G,$$

$$G=\frac{1}{hl}(1+n)\mu_0\left[\frac{1}{\mu_0^{1/n}}-1+210\,\gamma\left(\ln\frac{1}{\delta_0}-\frac{1}{n}\right)\right], \tag{28}$$

$$\nu_1=\begin{cases}\dfrac{1}{2-n}\left(1-\dfrac{2}{n}\delta_0^{2-n}\right) & \text{for}\quad n\neq 2,\\[2ex] \ln\dfrac{1}{\delta_0}-\dfrac{1}{2} & \text{for}\quad n=2.\end{cases}$$

If the coefficient q has an exponential temperature dependence,

$$\zeta(\theta)=\exp(-m\theta),\ \mu_0^{1/n}=\frac{n}{m}\left[1-\exp\left(-\frac{m}{n}\right)\right],$$

$$\beta=\frac{1}{2}B\left(1-2\varkappa\varepsilon+2\frac{n}{m}\right)-\frac{1}{6},$$

$$\gamma=2\left(\frac{n}{m}\right)^2\left\{B\left[1+60\left(\frac{n}{m}\right)^2\right]-10\frac{n}{m}\right\}, \tag{29}$$

$$B=2\frac{n}{m}\left(\frac{1}{\mu_0^{1/n}}-1\right)-1.$$

Here

$$m=b(T_0-T_f)=\ln\frac{q_f}{q_0}, \tag{30}$$

and b is a physical constant. For example, from data in [5] for pure vapor oil, we can take b = 0.15 1/deg, but for benzene with 82% resin, b = 0.06 1/deg.

In the limiting case $\Omega\to\infty$ and an arbitrary function $\zeta(\theta)$, we have

$$\theta=\theta_1=\begin{cases}0 & \text{for } \eta<\varkappa\varepsilon,\\ 1 & \text{for } \eta>\varkappa\varepsilon,\end{cases}\quad f=f_1=1-\varkappa\varepsilon,$$

$$L^*=\mu=[\varkappa\varepsilon+\zeta^{1/n}(1)(1-\varkappa\varepsilon)]^n. \tag{31}$$

If $m \ll 1$, the temperature dependence of q is linear, $\zeta = m\theta$, and the solution is written in the form

$$\mu=1-mf,\ L^*=1-m(1+n)\int_0^1 f\delta^{*n}d\delta^*. \tag{32}$$

By substituting the value of f from (27), we obtain L* for small Ω and 1/hℓ:

$$L^*=1-\frac{m}{2}\left\{1+\frac{1}{24}(1+n)\Omega(1-2\varkappa\varepsilon)[1-(1-n)\nu_1]+\right.$$
$$\left.+\frac{1+n}{nhl}\left(1-\frac{1}{n}+\ln\frac{1}{\delta_0}\right)\right\}. \tag{33}$$

This result can be obtained from Eq. (28) by noting that in this case

$$\mu_0=1-\frac{m}{2},\quad \beta=\frac{m}{12n}(1-2\varkappa\varepsilon),\quad \gamma=\frac{m}{420n}.$$

For arbitrary values of the parameters Ω, hℓ, and m, the problem can be solved by numerical integration. As a first approximation to the function $\mu(\delta^*)$, we take its average value in the interval

$$\mu_{(1)}=\frac{1}{2}[1+\zeta(1)]. \tag{34}$$

We calculate the corresponding temperature distribution $\theta_{(1)}$ from Eqs. (11), (13), (15), (16), and (18) and find the next approximation

$$\mu_{(2)}=\left[\int_0^1 \zeta^{1/n}(\theta_{(1)})\,d\eta\right]^n \tag{35}$$

etc., until the following approximation coincides with the previous one. Then we find L* from (20). Table 1 shows the result of such a calculation for $\kappa\varepsilon = 1/6$, $h = \infty$, and an exponential temperature dependence of the coefficient q. The computation process converges quickly and we take $\mu_{(2)}$ for the final function $\mu(\delta^*)$, because it coincides with $\mu_{(3)}$ to six significant figures.

Analysis of Table 1 shows that Eq. (28) can be used for $\Omega \leq 1$. Its relative error increases with n and Ω and reaches 3.6% for n = 2 and Ω = 1. Equation (31) is correct for Ω > 10, and Eq. (33) is correct for $m \leq 0.5$. The maximum error of the latter is 4% for m = 0.5.

The effect of heating the filler along the working length of the apparatus is characterized by the dependence of L* on m, because according to (30) m determines the heating temperature $T_0 - T_f$. With no heating, m = 0 and L* = 1. For m = 0.5 ($q_0 = 0.61\ q_f$), L* decreases by 20-34%, but for m = 1 ($q_0 = 0.37\ q_f$), by 34-55%. The dependence of L* on Ω is much weaker. As n changes from 0.5 to 2, the decrease in L* does not exceed 11%, and changing Ω from 0.01 to 100, L* decreases by 27%; therefore, the effect of these parameters on the length L depends basically on the multiplier L_f. According to the definition of Ω, the

TABLE 1. Values of L* for $\kappa\varepsilon = 1/6$ and $h = \infty$

n	m	Ω: 0,01	0,1	1	10	100
0,5	0,5	0,795	0,794	0,789	0,746	0,687
	1	0,657	0,656	0,646	0,573	0,503
1	0,5	0,787	0,786	0,774	0,708	0,669
	1	0,632	0,630	0,607	0,500	0,462
1,5	0,5	0,783	0,780	0,756	0,687	0,666
	1	0,622	0,616	0,570	0,471	0,452
2	0,5	0,781	0,773	0,736	0,678	0,664
	1	0,616	0,601	0,536	0,459	0,448

TABLE 2. Values of L^* for $\kappa\varepsilon = 1/6$ and $\Omega < 1$

n	m	hl		
		50	100	150
0,5	0,5	0,036	0,022	0,016
	1	0,060	0,036	0,026
1	0,5	0,031	0,018	0,013
	1	0,051	0,030	0,021
1,5	0,5	0,024	0,014	0,010
	1	0,039	0,023	0,017
2	0,5	0,017	0,009	0,007
	1	0,038	0,022	0,016

multiplier L_f is proportional to the drawing velocity u and inversely proportional to the pressure drop $p_f + p_c - p_0$ divided by the value 1 + n.

According to (28), external heat transfer decreases L^* by the value G. This is caused by a higher temperature in region III, because the fluid entering the filler now has a temperature higher than T_f. As can be seen from Table 2, the magnitude of G increases with increasing m and decreases with increasing n and Ω.

NOTATION

$\rho_{f,s}$ and $c_{f,s}$ are the density and heat capacity of the binder and the filler; ε is the porosity; λ is the thermal conductivity; h is the heat transfer coefficient; p_0 and p_f are the pressures in the pores of the filler and in the fluid at temperatures T_0 and T_f, respectively; p_c is the capillary pressure; v_y is the impregnation rate; u and 2ℓ are the drawing rate and the thickness of the filler; L is the working length of the impregnating apparatus.

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